The Measurement of the Permanent Magnetization of Rocks

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I. THE MEASUREMENT OF THE PERMANENT MAGNETIZATION OF ROCKS

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[Plate 1]

The adaptation of the astatic magnetometer to the measurement of the permanent magnetization of igneous and sedimentary rocks is described. The theory of the effect of non-uniform intensity of magnetization is given and the method of allowing for it is illustrated.

CONTENTS

1. Introduction

The development of the technique of measuring the weak permanent magnetization of sediments and the survey of the palaeomagnetism of certain sediments in the geological column in Great Britain, which form the subject of this series of papers, was undertaken in this Department in 1951. That the magnetization of igneous rocks might serve as a method of determining the direction of the geomagnetic field in remote geological epochs had long been known. That a similar possibility existed for sediments had recently been discovered, but widespread scepticism existed in the value of the fragmentary results obtained thus far. In particular, the phenomenon of reversed magnetization was attracting great attention, largely through the work of Hospers (1953, 1954) on the Tertiary and Quaternary basalts of Iceland, and it was desirable to examine whether reversals occurred in other geological periods and in sedimentary strata.

The development by Blackett (1952) of a sensitive astatic magnetometer provided the incentive to extend this survey to the more weakly magnetized sedimentary rocks, and this paper is concerned with the adaptation of this instrument to the practical problem of the determination of the intensity and direction of magnetization of weakly polarized specimens.

2. General theory of the design

An astatic magnetometer is essentially a magnetic gradiometer. The magnet system shown in figure 1, plate 1, consists of two parallel, horizontal and oppositely polarized magnets of nearly equal moment P. These are rigidly connected a distance L apart, and the whole is suspended vertically by a phosphor-bronze strip. If the difference at the magnets between the horizontal field components at right angles to their axes is h, a deflexion (θ) of the suspended magnet system will occur, the sensitivity s being given by

$$s = \theta/h = T^2P/4\pi^2\alpha I_0, \tag{2·1}$$

where I_0 and αI_0 are the moments of inertia of one magnet and of the whole system respectively about the vertical axis, and T is the period of oscillation in the absence of a field gradient. There is a desirable upper limit for the response time in order to make measurements quickly and to minimize errors due to drift. For the greatest sensitivity for a given period the ratio P/I_0 must be made as large as possible. The smaller the magnet system the greater the ratio P/I_0 for given materials of a given shape, but mechanical and optical difficulties set a lower limit to the size. P/I_0 also depends on the magnet material and shape, and Alcomax IV and Ferroxdure magnets of the shape and size given in table 1 are considered to possess the optimum characteristics. l and b are the lengths of the sides of the magnets parallel and perpendicular respectively to their direction of magnetization. The ratio of l to b is termed β.

The r.m.s. deflexion θ_0 of the magnet system due to thermal noise is given by

$$\tfrac{1}{2}C\theta_0^2 = \tfrac{1}{2}kt,$$

where C is the torsional constant of the suspending fibre, k is Boltzmann's constant and t the absolute temperature.

The thermal deflexion given in the table is the linear deflexion at the scale corresponding to θ_0. The minimum detectable field is defined as the magnetic field difference which will produce a deflexion equal to the r.m.s. thermal deflexion.

TABLE 1. PARTICULARS OF MAGNETOMETERS

dimension of quantity	Alcomax IV system	Ferroxdure system
I_0 (c.g.s.)	0·0044	0·0044
α	2·5	2·5
L (cm)	10·0	10·0
l (cm)	0·6	0·29
b (cm)	0·15	0·74
P (G cm^3)	9·5	15·0
$P/I^{\frac{1}{2}}$ (c.g.s.)	45	70
β	4·0	0·4
sensitivity (G mm^{-1} at 5 m) for $T = 40$ s	$2·4 \times 10^{-9}$	$1·8 \times 10^{-9}$
for $T = 15$ s	$1·8 \times 10^{-8}$	$1·3 \times 10^{-8}$
thermal deflexion (mm at 5 m)	0·12	0·12
minimum detectable field (G)	$2·9 \times 10^{-10}$	$2·2 \times 10^{-10}$
cross-section of suspension strip (in.)	$0·001 \times 0·0001$	
C (dyn cm rad^{-1})	0·277	
length of suspension (cm)	20	

In order that the maximum theoretical sensitivity should be attained in practice, it is important that the control should be purely torsional. The magnetic control due to the geomagnetic field is reduced by giving the magnet system a high astaticism (S) and by reducing the horizontal component of the field with a compensating coil system. The upper limit of astaticism which is retained for periods of several months is about 5000, though values of S up to 30000 have been obtained for shorter periods. If the horizontal component H of the geomagnetic field is also reduced by a factor S', then HP/SS' must be made less than C, the torsional constant of the suspension.

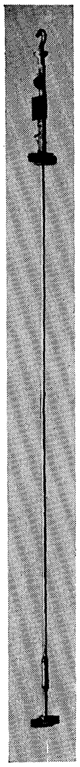


FIGURE 1. Magnet system, consisting of astatic pair, trimming magnets and mirror.

FIGURE 3. Magnetometer with Fanselau and Helmholtz coil systems.

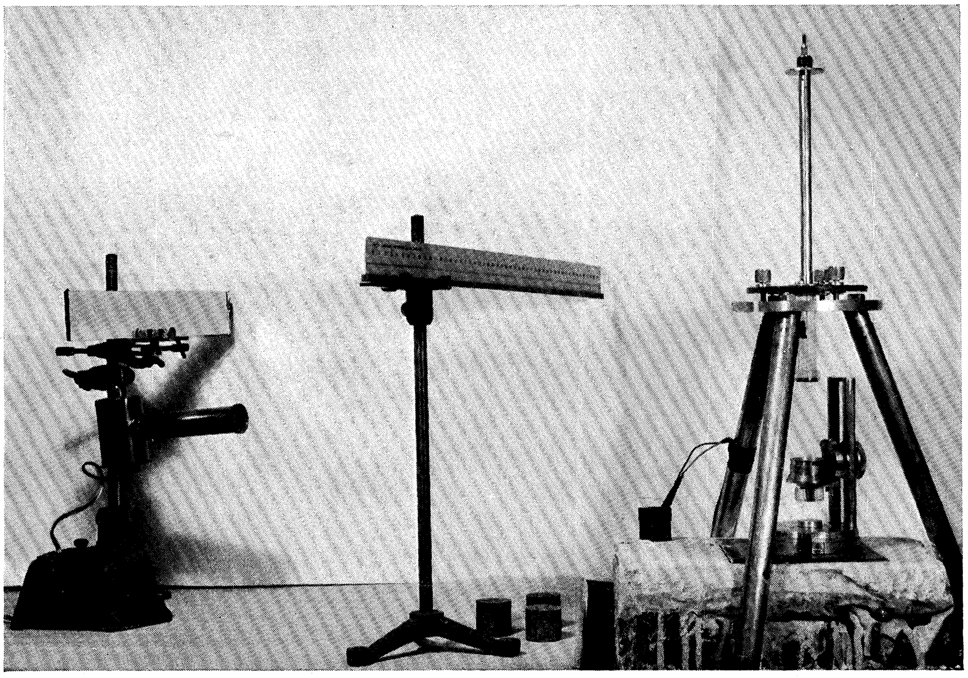


FIGURE 4. Lay-out of short-period magnetometer.

With an astaticism of 5000, 80 % of the full sensitivity is obtained without compensating the horizontal component of the earth's field, though this is desirable for other reasons, principally that the rock specimen should not have induced magnetization. For this purpose also a Helmholtz coil system is employed to compensate for the vertical component of the geomagnetic field.

Gradients in the residual field must be negligible. These are produced by ferromagnetic impurities in the instrument case and by the compensating coil system. The instrument case was made of laminated beechwood, as is shown in figure 2, and the rest of the instrument of Perspex and Duralumin, shown in figure 3, plate 1. Additional precautions were taken concerning the materials used for the concrete base and the walls of the hut in which the magnetometer was installed. If the horizontal compensating coils produce a small field difference of h' at the magnets, $h'P\theta$ must be less than C, where θ is the deflexion of the system from its null position parallel to the axis of these coils. Since θ is small it is not difficult to satisfy the above condition. However, if the magnet system is slightly offset from the axis of the vertical compensating coils, these produce slight horizontal components which tend to rotate each magnet in the same sense. If h'' is the field difference then $h''P$ must be less than C, a condition difficult to obtain if the diameter of the compensating coils is too small since h'' is inversely proportional to the fourth power of the radius of the coils. If a Helmholtz pair is used, the radius of the coils must be about 120 cm, since the accuracy with which the magnet system can be centred along the axis of the vertical field compensating coils is about 1 mm. It was preferred, however, to use a four-coil system, and one designed by Fanselau (1929) was chosen. It consists of two pairs of coils of radii a_1 and a_2 and separation d_1 and d_2 respectively, the relative dimensions being given in table 2. The radius a_2 was 60 cm.

TABLE 2. DIMENSIONS OF FANSELAU SYSTEM

d_1/a_1	d_2/a_1	a_2/a_1	d_2/a_2
1·107	0·364	1·309	0·278

3. DETAILS OF THE CONSTRUCTION OF THE MAGNETOMETER

Figure 1 shows the Alcomax IV system. The collar permitting relative rotation of the magnets about a vertical axis is near the lower end of the Duralumin stem. A small mirror 4 mm square is placed above the top magnet and between the two heads holding the trimming magnets, one of which can be rotated in the plane of the main magnets and the other in the vertical plane at right angles. At the top of the holder is the hook to which is attached the suspension strip, and also a small cone which raises the magnet system when the jaws of the clamping device close, thus relaxing the tension in the fibre.

Figure 2 shows the magnetometer case with the lower part removed to reveal the magnet system, held by the clamping device. The torsion head, which fits into the upper part of the case, carries the phosphor-bronze suspension strip. The lower part of the case, seen in figure 3, contains the viewing window. A copper damping plate is attached to the case by a threaded holder by which its distance from the lower magnet can be adjusted.

The two parts of the magnetometer case are fitted together by a bayonet-type socket. The entire case rests kinematically on the tripod table, and the magnet system is made to hang centrally in the case by means of the levelling screws. The tripod is adjustable in the horizontal plane for centring within the coil system.

The nature of the measurements made require the specimen to be raised and lowered below the magnet system in a time short compared with the period of oscillation, rotated to different azimuths, and traversed horizontally in a direction at right angles to the magnet system. For convenience of measurement, and because of the thermal and magnetic disturbances produced when the instrument is approached, these movements are remotely controlled from a reading desk.

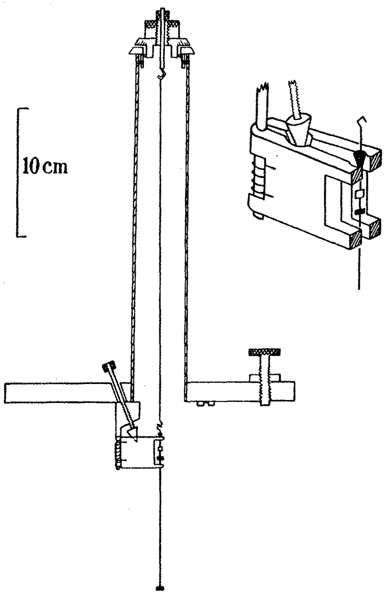


FIGURE 2. Instrument case with lower part removed.

A horizontal table runs kinematically in the vertical guides and is raised and lowered by a cord and pulleys. The height is controlled by means of a stop, threaded on a rod which can be turned by the lower of two pulleys in the base of the mounting. The distance of the specimen below the magnetometer may thus be repeated accurately for each measurement, the height being read by means of a vernier and scale. Sliding horizontally between guides on the table is a base which supports a rotating head. The base is spring-loaded to bear against a cam, which can be rotated about a vertical axis. With the table in the lowered position the cam engages with a dog attached to the upper of the two base pulleys. Nine steps are cut on the cam, corresponding to a central position under the magnetometer, and four positions on either side, each step being a displacement of 0·25 cm.

The height of the table and the azimuth and traverse (off-centre) positions are read with the aid of a telescope and lenses from the reading desk. The traverse pointer, attached to the underside of the azimuth lens, moves over a scale engraved on the side of the horizontal table.

The specimen is carried in a self-locating holder which is loaded outside the magnetometer room, and placed in position on the mounting with a rod through a small door in the wall.

The vertical guides on which the table moves are rigidly supported in the Duralumin frame, the position of which can be accurately adjusted beneath the magnetometer.

4. ADJUSTMENT AND CALIBRATION OF MAGNETOMETER

(a) *Adjustment of the coil systems*

The coil system compensating the horizontal field was set in the meridian and the current required to null the horizontal component of the geomagnetic field determined with a vibration magnetometer. The current required to null the vertical component of the geomagnetic field was determined with a Kew pattern dip circle.

(b) *Methods of astaticizing the magnet system and measuring the sensitivity*

Several pieces of sintered Alcomax IV and of Ferroxdure were cut and ground to the required shape. Those of approximately the same weight were magnetized to saturation, slightly aged in an a.c. field and the best pair of magnets of each material selected. Each pair was then placed in a magnet holder and the lower magnet rotated in the stem collar until the magnetic axes were anti-parallel, when the period of oscillation was a maximum. By slightly demagnetizing the stronger magnet in a weak a.c. field, an astaticism of 500 was easily obtained and occasionally 1500 was reached. The residual moment of about 0·03 c.g.s. was further reduced by means of the short rotatable Vicalloy wire trimming magnets, the final positions of which were fixed with shellac. The astaticism was calculated from the deflexion produced by a uniform horizontal field.

The sensitivity of the instrument, defined here as that field difference required to produce 1 mm deflexion on a scale 5 m distant, was determined by measuring the deflexion due to a known field gradient produced by a small coil vertically above the magnetometer.

5. DESIGN OF A VERY SHORT-PERIOD MAGNETOMETER

For the measurement of igneous rocks and very strongly magnetized sediments of intensity about 10^{-4} G, an instrument having a time constant of the order of 1 s has been constructed. For a short period, I_0 should be as small as possible, and the condition that P/I_0 should be a maximum is no longer so important. Alcomax IV magnets of fineness ratio equal to 2 were used.

With such a short period, the direction of magnetization of a cylindrical specimen can be determined by rotating it, first about a vertical axis until maximum deflexion is obtained and then about a horizontal axis parallel with the magnetic axis of the suspended system, until the zero deflexion is obtained. Then the magnetization vector is vertical and its direction with respect to orientation lines on the specimen can be read directly.

It is important to fix the specimen holder to a pillar separated from the tripod carrying the magnetometer so that vibrations due to touching the specimen are not transferred to the instrument which is shown in figure 4, plate 1. Particulars of the instrument are listed in table 3.

6. METHODS OF COLLECTION

The strike direction and dip of a surface on the rock are determined in the usual way and recorded. The geological strike and dip are also determined to allow a correction to be made to relate the measurements to the original position of the rock on formation. If the bedding surfaces are exposed, as is often the case, it is convenient to record on these, for the dip of the strata is then given by the same measurement.

Table 3. Particulars of a very short period magnetometer

length of magnets (l) (Alcomax IV)	3 mm
breadth of magnets (b)	1·5 mm
magnet mass (M)	$5{\cdot}07 \times 10^{-2}$ g
magnetic moment (P)	2·3 G cm^3
moment of inertia of magnet system (αI_0)	$1{\cdot}2 \times 10^{-3}$ c.g.s.
sensitivity	$1{\cdot}1 \times 10^{-6}$ G/mm at 1 m
time constant (T)	3 s
degree of astaticism (S)	500
suspension strip (phosphor-bronze)	0·005 × 0·0005 in.
length of suspension	16 cm
torsional constant of suspension strip (C)	21·7 dyn cm rad^{-1}
magnetic restoring force due to geomagnetic field	$8{\cdot}2 \times 10^{-4}$ dyn cm rad^{-1}

Two or three cores $1\frac{3}{8}$ in. in diameter are usually cut from each specimen with a diamond-impregnated trepanning tool mounted in a vertical drill. From these, five or more disks 5 to 8 mm thick were in most cases cut such that the plane of the disks was parallel to the bedding planes. This simplifies the procedure, automatically correcting for geological dip, and in the discussion it will be assumed.

7. MEASUREMENT OF THE DIRECTION OF MAGNETIZATION OF SEDIMENTARY ROCKS

Suppose that the axes of cylindrical co-ordinates (z, r, θ) are fixed in the disk such that the direction Oz was the upward direction of the vertical and $\theta = 0$ the present geographical north at the time of the formation of the rock. Assuming the disk to be uniformly magnetized, the horizontal component of the magnetic intensity M_H produces a horizontal field on the axis in the direction θ given by

$$h = vM_H \cos(\theta - D)/z_0^3, \tag{7·1}$$

where v is the volume of the disk and $z_0^2 = a^2 + z^2$, where a is the radius of the disk. The vertical component M_z of the intensity is equivalent to a spherical magnetic shell, bounded by the periphery of the disk, of strength $M_z t$, where t is the thickness of the disk. Thus at a distance z along its axis, the potential is $vM_z(z_0 - z)/az_0$, and as the field is solenoidal,

$$\frac{\mathrm{d}h}{\mathrm{d}r} = \frac{1}{2}\frac{\mathrm{d}h_z}{\mathrm{d}z} = \frac{3vM_z}{z_0^4}\frac{z}{z_0}, \tag{7·2}$$

where h_z is the vertical component of the field. The procedure of measurement is shown in figure 5. The on-centre curve is the deflexion of the magnetometer, when the axes of the disk and magnet system coincide, plotted against the azimuth θ, which is the angle between the plane of the magnets and the plane $\theta = 0$ in the disk. The phase of this curve gives D. From the amplitude d_0 of the curve, M_H can be determined:

$$M_H = sd_0 z_0^3/v. \tag{7·3}$$

Similar curves are obtained when the axis of the disk is displaced a small distance ($\mathrm{d}r$) to either side of the axis of the magnet system. The vertical displacement d of this curve from the previous one gives M_z,

$$M_z = sz_0^5\, d/3vz\mathrm{d}r. \tag{7·4}$$

On inverting the rock disk in the 'on-centre' position and measuring θ in the opposite sense, the same curve should be obtained, as, for example, in figure 7. It is commonly found, however, that the 'upright' and 'inverted' curves, though sinusoidal have different amplitudes and phases, as in figure 6, and that the vertical displacements of the curve when the disk is moved to the W and E positions in its upright and inverted positions are unequal as in figure 5. This must arise from non-uniformities in the intensity of magnetization, the effects of which can be approximated by quadrupoles at the centre of the disk.

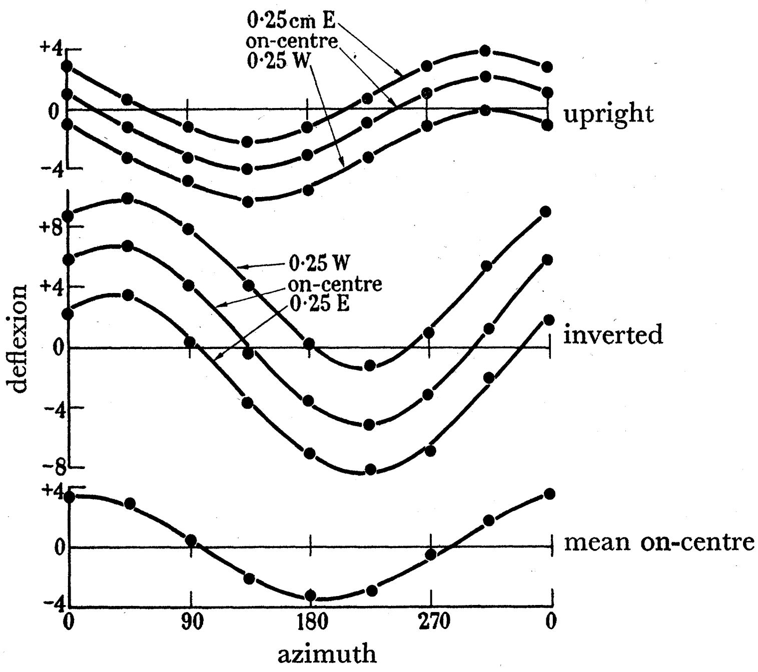
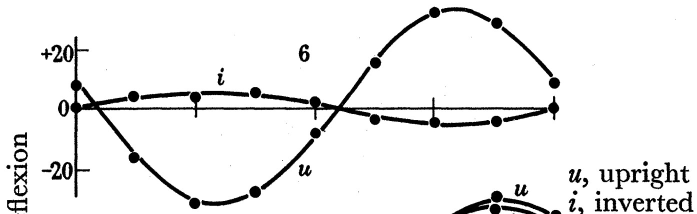


FIGURE 5. Magnetometer deflexion plotted against azimuth (θ) for upright and inverted positions of a disk of Torridonian sandstone. Mean curve is also shown.



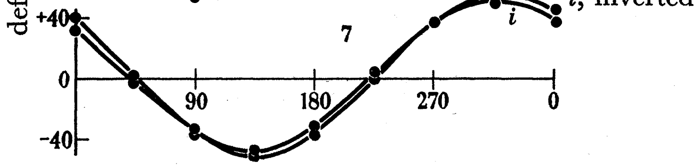


FIGURE 6. Magnetometer deflexion plotted against azimuth (θ) for Torridonian sandstone disk with non-uniform intensity of magnetization.

FIGURE 7. Magnetometer deflexion plotted against azimuth (θ) for Torridonian sandstone disk with uniform intensity of magnetization.

The horizontal field on the axis Oz due to these will be reversed when the disk is reversed, and is thus eliminated in the mean of the upright and inverted curves. Similarly, the effect on the horizontal gradient of the horizontal field near to the axis can be removed by averaging the displacements d on either side of the axis and in the upright and inverted positions.

Figures 8 and 9 show that formulas (7·1) and (7·2), when the quadrupole component is so eliminated, represent very well the magnetization of rock disks, provided that z exceeds about 3 cm. The dotted curves for specimen 7 show the effect of assuming that the mag-

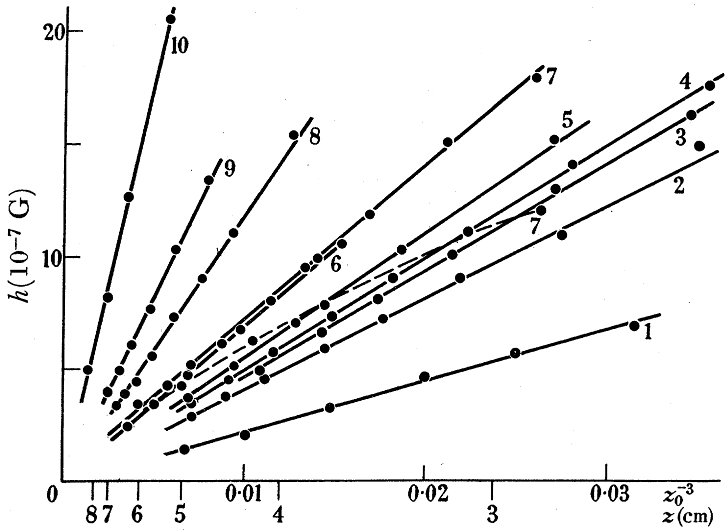


FIGURE 8. Variation with distance z of h for ten disks of Torridonian sandstone.

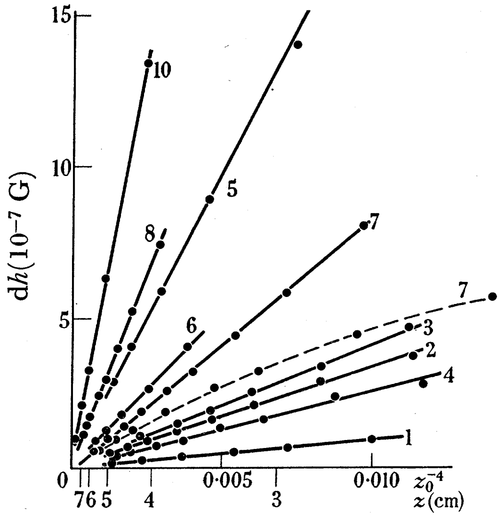


FIGURE 9. Variation with distance z of dh for ten disks of Torridonian sandstone.

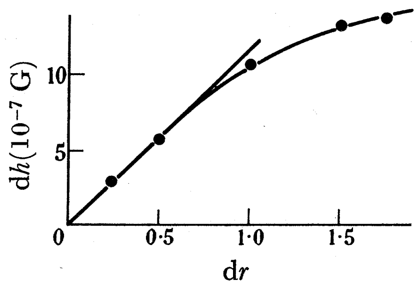


FIGURE 10. Variation of dh with dr for $z = 3·4$ cm for disk of Torridonian sandstone.

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netization is that of a dipole at the centre of disk, instead of using the above formulae (which effectively include an octupole component due to the shape of the disk).

Figure 10 shows that the variation of the field off-centre at the working distance is linear until dr exceeds about 0·5 cm.

8. Measurement of quadrupole components

Figure 11 demonstrates that the effect of non-uniform magnetization is to introduce a quadrupole component. The effect of this component will decrease inversely as the fourth power of z. If h, dh and D are calculated from formulas (7·1) and (7·2) for the upright and inverted positions, their differences Δh, $\Delta \mathrm{d}h$ and ΔD should decrease with z, the first two linearly. Figure 11, in which Δh and $\Delta \mathrm{d}h$ are expressed as percentages of the mean h and dh, shows this to be true.

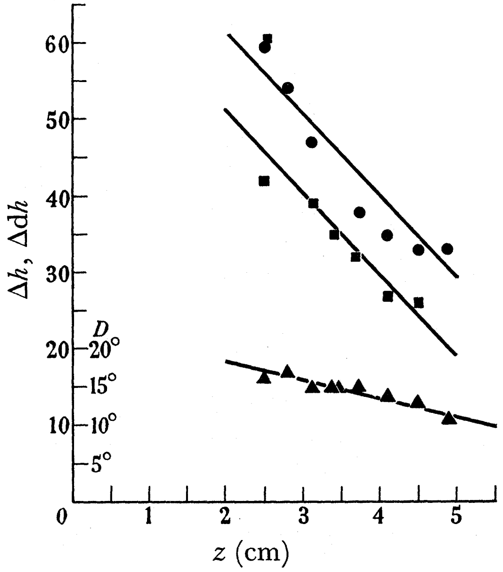

Figure 11. Variation of Δh, $\Delta \mathrm{d}h$ and D with z for disk 2.

Table 4. *Fields due to quadrupole components*

type of quadrupole		quadrupole moment	horizontal field components on the axis		radial component of horizontal field component near the axis
			x component	y component	
in the plane of the disk	axial (along Ox axis)	q_{xx}	0	0	$3q_{xx}(1+2\sin^2\theta)\,\mathrm{d}r/2z^5$
	transverse (symmetrical about Ox and Oy)	q_{xy}	0	0	$3q_{xy}\sin 2\theta\,\mathrm{d}r/2z^5$
along the axis of the disk	axial	q_{zz}	0	0	$6q_{zz}\mathrm{d}r/z^5$
	transverse (in Oxz plane)	q_{xz}	$3q_{xz}/2z^4$	0	$15q_{xz}\cos\theta(\mathrm{d}r)^2/2z^6$

The contributions to h and dh from quadrupole components are listed in table 4; in the final column the first term in (dr/z) only is included. It will be seen that the component q_{xz} contributes to the horizontal field on the axis, consequently producing a difference between the upright and inverted curves in both amplitude and phase. The component q_{xy} and, to a lesser degree, the component q_{xx} produce a variation with azimuth in the differences between the curves off and on centre. Apart from this variation, these differences in the upright and inverted positions will not be the same because of the contributions of the components q_{xx} and q_{zz}.

9. MEASUREMENT OF SUSCEPTIBILITY

The magnetization, induced by weak fields of the order of 0·5 G, can readily be measured by this magnetometer. The fields are applied to the specimen by the Fanselau and Helmholtz coil systems; this causes reduction in sensitivity, though not by a large factor.

It is occasionally found that certain rocks when measured by the methods of § 7, at $z < 3$ cm, give non-sinusoidal curves (see figure 12). These cannot arise from their permanent magnetization or from a uniform susceptibility. Certain green siltstones from the Torridonian series and grey-green mudstones from the Silurian of Westmorland with a high susceptibility of the order of 10^{-4} and a low remanent moment show this effect. It is likely that this results from the induction produced in a non-uniform sample by the field of the lower magnet.

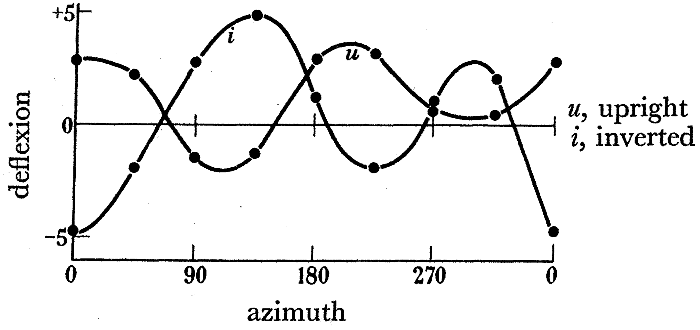

FIGURE 12. Magnetometer deflexion plotted against azimuth for disk of green siltstone from the Torridonian series.

10. ANALYSIS OF DIRECTIONS OF MAGNETIZATION

The declination D, inclination I and intensity M of each disk was thus determined, of which the intensity is not simply related to the ancient geomagnetic field. Thus the D and I of all the N disks obtained at one site are each regarded as unit vectors. The sum R of these gives the mean declination and inclination D_s and I_s. Following the method of Fisher (1953) this is regarded as a sample of a population the frequency distribution of which is proportional to exp ($\kappa \cos \theta$), where θ is the angular divergence from the true direction. An estimate of the precision κ_s is given by $(N-1)/(N-R)$. A cone of confidence, of semi-angle α_s, described with the mean direction as its axis includes the true direction with a probability of 95 %. The mean declination and inclination at a number of sites may be similarly treated to provide a mean declination and inclination D_m and I_m for a series of rocks, with radius of confidence α_m and precision κ_m.

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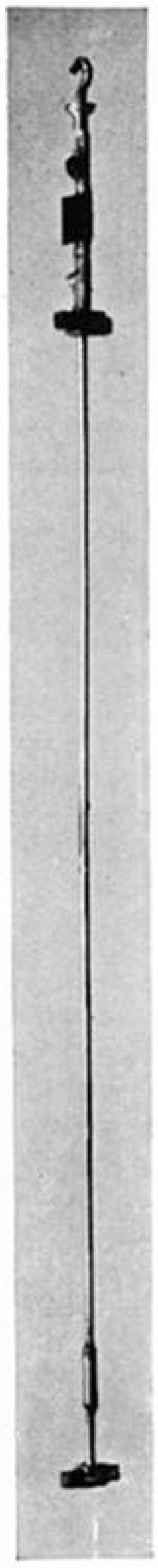
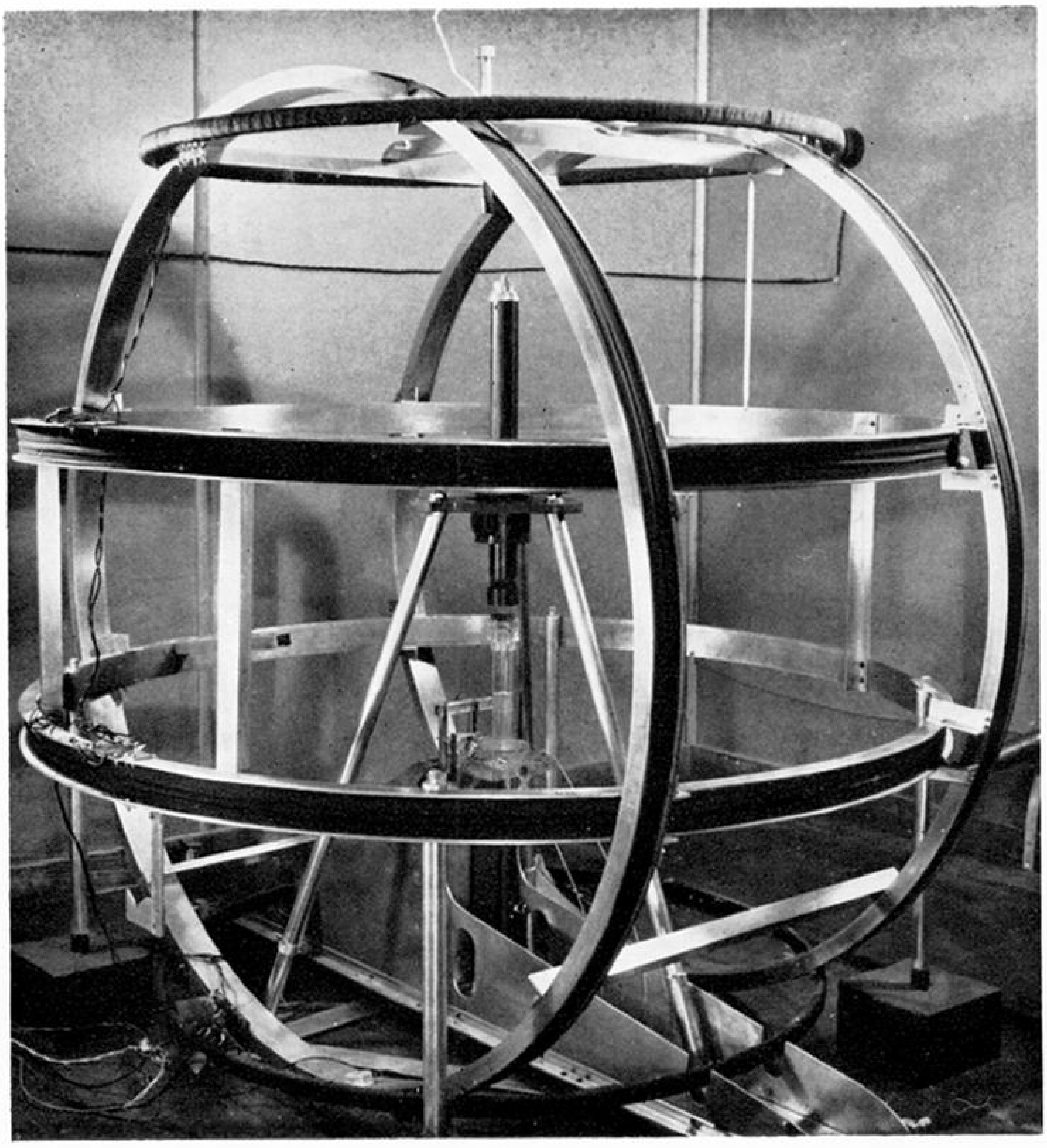


FIGURE 1. Magnet system, consisting of astatic pair, trimming magnets and mirror.



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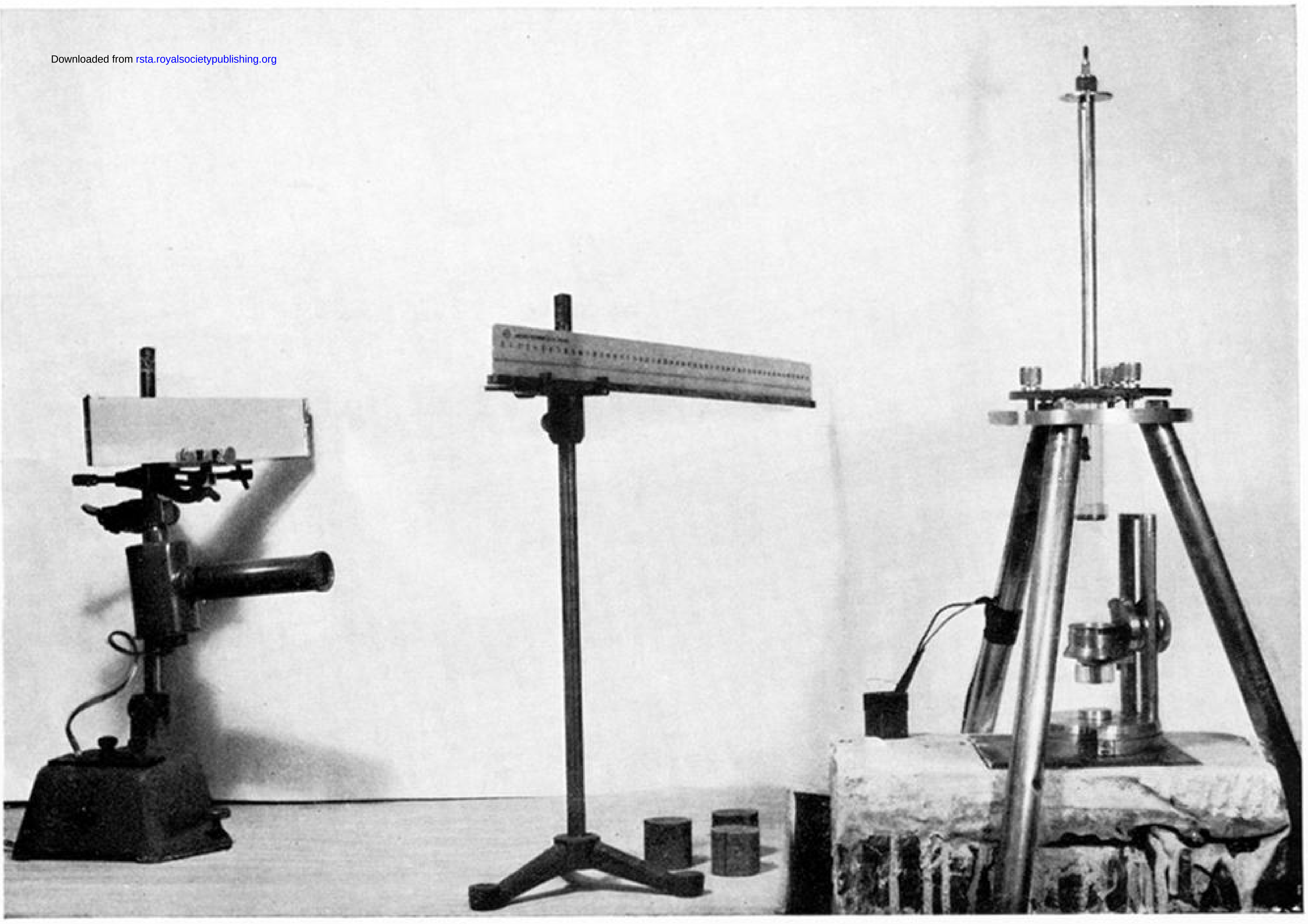

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